Clarence Barlow

On the Quantification of Harmony and Metre

1. On Metre

[Tape example]

This piece is called *Otodeblu*. It was played on a Roboard, a pianola driven by a computer. It was composed in 1990 using my programme Autobusk here on this Atari computer; it runs on various parameters such as tonality (making the music more - or less tonal, as the case may be), metricity and such like. Let me give you an example of how the programme works; then I'll explain why I needed a programme to compose the piece.

When I start the programme, you will hear some random music which is produced as a result of probability tables; let's just start it and listen [starts computer programme]. The music is set to a very clear key and to a very clear metre. Let me now very slowly take down both the metricity and the tonality [operates programme].... Ok, we've finally reached a stage where there is no feeling of key or metre any more. There is of course a pulse feeling, but it isn't graduated any more into the form of a really clear metre - where you could tell where the "one" would be.

Why does one need this? Suppose you want to write a music which is (as I've done here) very variable in both its key and its metre feeling. You need to know how to move to intermediate stages. What for example would be an adequate definition of metric music as opposed to non-metric music? For this I chose a very simple algorithm or definition: In the case of ametric music, all the pulses are equally probable.

So no matter what metre you have, suppose six or eight beats in the bar or whatever, they will in this case all have the same probability. Which means the bar doesn't make any more metric sense. But if you want to make the music more and more metric, you have to then decide how probable or how important the individual pulses ought to be. This assumes there might be a correlation between their importance and their probability.

Fig.1 - Two metres compared

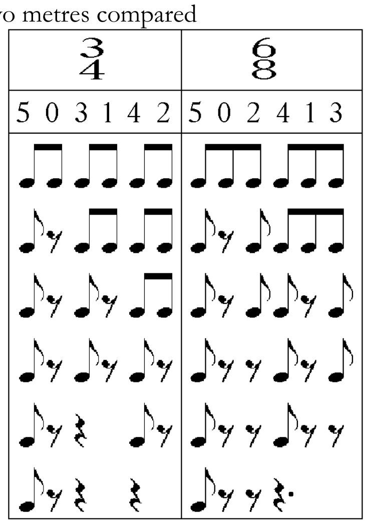


Fig.1 shows an example of two metres, 3/4 and 6/8. If we look at these rhythms you'll find them getting gradually thinner: in each case I've taken away one attack at a time. And I think you'll agree with me that the right column goes much more clearly together with a 6/8 feeling, and the left with a 3/4 feeling. This is reflected in a series of numbers at the top - [5,0,3,1,4,2] and [5,0,2,4,1,3], an ordering of the individual pulses according to their importance. I call this the *Indispensability of Attack*. The formula for it is somewhat threatening - this is only its main part (see Fig.2)[1] - but you can programme it into a computer and then forget it.

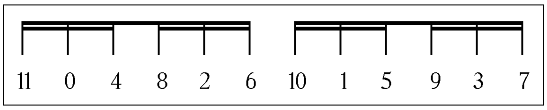
According to this system, any metre can be expressed in terms of the relevance of its pulses.

Fig.2 - Indispensability formula

$$\psi_z(n) = \sum_{r=0}^{z-1}\left\{ \prod_{i=0}^{z-r-1} q_i \Psi_{q_{z-r}} \left(1 + \left(\left[1 + \frac{(n-2) \bmod \prod_{j=1}^{z} q_j}{\prod_{k=0}^{r} q_{z+1-k}} \right] \bmod q_{z-r} \right) \right) \right\}$$

Let me give you one more example. Take a 12/16 measure (see Fig.3): this system numbers the indispensability from 11 down to 0 in the given order.

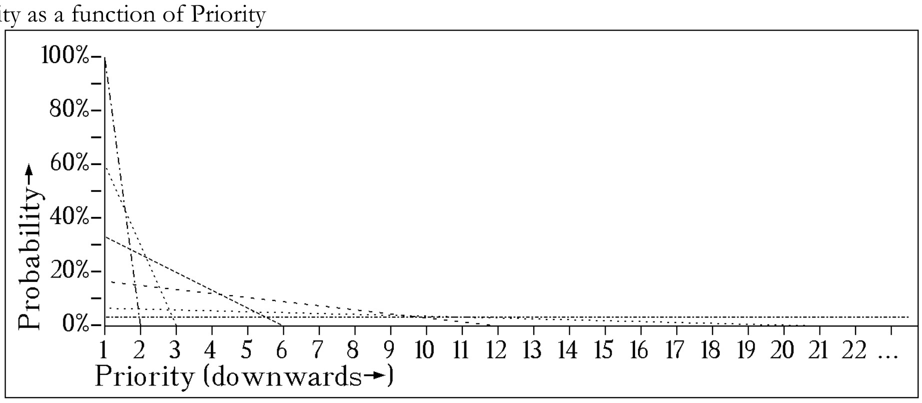
Fig.3 - 12/16 indispensabilities



2. On Priority and Probability

What do I do with these priorities? I set up a correlation between priority on the X-axis and probability on the Y-axis, as in Fig.4. In the case of an ametric music all pulses are equally probable, irrespective of priority - see the horizontal line. But the more metric the music gets, the steeper the line. This means that in this case the higher the priority (here 1 is higher than 2), the higher the probability. This very simple relation allows one to compose a music that can vary smoothly from metric to ametric.

Fig.4 - Probability as a function of Priority



Now I must say at this point that I am most decidedly not a musicologist, but a composer in search of techniques to realise certain pieces. Of course, the discoveries I make, the techniques I develop are very often close to things that seem to have some musicological significance. But I leave that to you. What I say here concerns finding a tool for making a piece of music.

The same correlation of priority and probability goes for the pitch domain. Using a pitch set (this could be e.g. a major or a chromatic scale), I allot to each pitch a certain unique priority, no two pitches the same way. And if the music is to be tonal, pitches of higher priority become more probable and thus more frequent.

There is also a correlation between the pitch and pulse domains (see Fig.5). Where the pulse is important, more indispensable, tonality rises, going down on the weaker pulses. This is because you can effect smooth motion in a music by passing notes on weak beats and structural notes (Schenker's term) on strong beats.

Fig.5 - Pitch-Pulse matrix for a Major Scale and a 6/8 Metre

MAJOR SCALE	Pitch:	SIX-EIGHT METRE					
		Pulse: 1	2	3	4	5	6
		Priority: 1	6	4	2	5	3
C	2	30.4	12.5	12.6	15.3	12.5	13.1
B	7	0.0	12.5	12.4	9.7	12.5	11.9
A	6	0.0	12.5	12.4	10.8	12.5	12.1
G	3	19.6	12.5	12.6	14.2	12.5	12.9
F	8	0.0	12.5	12.3	8.5	12.4	11.7
E	5	0.0	12.5	12.5	11.9	12.5	12.4
D	4	8.8	12.5	12.5	13.1	12.5	12.6
C	1	41.2	12.5	12.7	16.5	12.6	13.3

So I raise the degree of tonality (reflected by the angle of the line in Fig.4) and decrease it wherever the pulse is weaker. Where do I get my pitch priorities? Now that's a different kettle of fish......

3. On Harmonicity

For several centuries people have been saying that musical intervals are of greater and lesser consonance. Another word used for that is *Harmonicity*. The word "consonance" I reserve for a totally different phenomenon: in the piano's middle range, a major second sounds generally more consonant than a minor second. But taking the same two intervals down to the bottom octave, you'll find the minor second the less dissonant of the two. Compare the perfect fourth and the tritone in the middle, then in the bottom range - the dissonance/consonance behaviour is reversed.

The sonic roughness caused by hairs on the basilar membrane and by other physiological matters is a phenomenon I call "consonance and dissonance", corresponding to general usage. It has to do with timbre and the basilar membrane, which I think Jim Tenney and Stan Tempelaars will tell us a lot more about, so I won't go into it any further. For me, "harmonicity" is the phenomenon which establishes whether intervals are more stable, like the octave and fifth, or less, like the tritone, which I call less harmonic.

It would seem logical to me that a music which is atonal uses all intervals equally probably or frequently. But if the music gets increasingly tonal, then more harmonic intervals, like octave and fifth, gain in probability against those of lesser harmonicity. The question now is: What is more and what is less harmonic?

Pythagoras was one of the first to say that the two numbers of an interval ratio are an indication of the (as I say) "harmonicity": the smaller the numbers, he said, the more harmonic the interval and vice versa. Hindemith, Schoenberg, Partch were to say the same thing several centuries later. But the concept of size alone is problematic - the intervals 1:2 (octave), 2:3 (fifth), 3:4 (fourth) etc. - are all nice and harmonic, and that's what we also learn at school. Going up to 5:6 that's fine, but the next interval 6:7 is generally not used in music of the West (or, by the way, of India). Nor is 7:8. But 8:9 is the well-known major tone, followed by the minor tone 9:10; another gap then ensues containing 10:11, 11:12, 12:13, 13:14, 14:15, none of which you'd normally find in classical music. 15:16, which comes after, is our minor second. One can see that all intervals in those gaps - 6:7, 7:8, 10:11, 11:12, 12:13, 13:14 and 14:15 - contain prime numbers larger than 5, the primes 7, 11 and 13 in this case. Thus the primeness of the factors plays a role as well: harmonic intervals are formed not only by small numbers but also by divisible numbers. Both are essential - smallness and divisibility.

In 1978 I came up with what I called the Indigestibility of Numbers, a concept combining smallness and divisibility. If a number is large but divisible, it is "digestible". Fig.6 shows the formula: the integer "N" is the product of powers "n" of primes "p"; put "n" and "p" into this formula and you get the indigestibility value expressed by the Greek letter ξ.

Fig.6 - Indigestibility formula

$$\xi(N) = 2\sum_{r=1}^{\infty}\left\{\frac{n_r(p_r-1)^2}{p_r}\right\}$$

Examples of this for 1-16 are to be found in Fig.7: the primes 7, 11 and 13 are very indigestible, their corresponding values (with that of 14) being the only ones over 10. The value 10 seems to be a kind of general cultural indigestibility barrier, since only less indigestible numbers are commonly used in classical music ratios. Note the power 2 in the formula; raising this makes higher primes much more indigestible, so this power is a key to the rate at which the indigestibility increases with the primes. I call it the "prime enmity factor", because the lower it is, the friendlier, i.e. more digestible each prime gets.

Fig.7 - Indigestibilities 1-16

N	ξ (N)
1	0. 0000000
2	1. 0000000
3	2. 6666667
4	2. 0000000
5	6. 4000000
6	3. 6666667
7	10. 2857143
8	3. 0000000
9	5. 3333333
10	7. 4000000
11	18. 1818182
12	4. 6666667
13	22. 1538462
14	11. 2857143
15	9. 0666667
16	4. 0000000

Using the above I now arrive at a formula for harmonicity (Fig.8) by adding the indigestibility values and inverting the sum.

Fig.8 - Harmonicity formula for an interval P:Q

$$H(P,Q) = \frac{\operatorname{sgn}\left(\xi(Q)-\xi(P)\right)}{\xi(P)+\xi(Q)}$$

The numerator tells you the direction of the interval's polarisation. For example, you might agree with me that the perfect fourth pulls upwards, that its upper note is the root. This is a matter of feeling; whether we agree or not is another matter. I here assume that e.g. the perfect fourth and the minor sixth have an upwards pull and that the fifth and the octave have a downwards one - the "sgn" sets a plus/minus sign for the polarity.

Here in Fig.9 is a set of intervals in one octave, going from 0 to 1200 cents (100 cents form one semitone). Given its ratio, each interval's harmonicity is derivable according to the formula in Fig.8; for example the perfect fifth 2:3 has a harmonicity of 0.2727, the perfect fourth 3:4 has a value of -0.2143 (upwards polarised) and so on. And so you can see that for various intervals you have varying degrees of harmonicity. All this is part of a composer's technique; you can get various harmonicity values which do seem to correspond pretty well to at least my feeling and to that of some colleagues of mine.

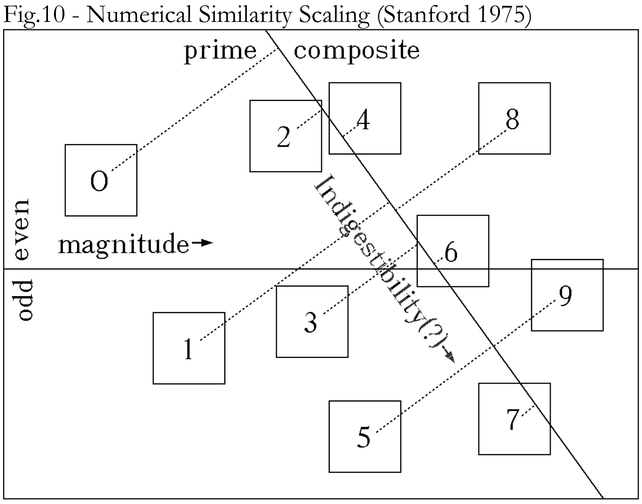
Fig.9 - Some Harmonicities

Cents	Ratio	Harmonicity
0. 000	1: 1	∞
111. 731	15: 16	−0. 076531
182. 404	9: 10	0. 078534
203. 910	8: 9	0. 120000
231. 174	7: 8	−0. 075269
266. 871	6: 7	0. 071672
294. 135	27: 32	−0. 076923
315. 641	5: 6	−0. 099338
386. 314	4: 5	0. 119048
407. 820	64: 81	0. 060000
435. 084	7: 9	−0. 064024
498. 045	3: 4	−0. 214286
519. 551	20: 27	−0. 060976
701. 955	2: 3	0. 272727
764. 916	9: 14	0. 060172
813. 686	5: 8	−0. 106383
884. 359	3: 5	0. 110294
905. 865	16: 27	0. 083333
933. 129	7: 12	−0. 066879
968. 826	4: 7	0. 081395
996. 090	9: 16	−0. 107143
1017. 596	5: 9	−0. 085227
1088. 269	8: 15	0. 082873
1200. 000	1: 2	1. 000000

Indigestibility is also to be found in interesting places. I asked friends of mine in a restaurant (which is a very nice place to ask friends) "if you had a round cake or a pizza, or something like that, to be cut into equal segments, what would be the easiest number of pieces to cut it into?" And they all said 2. And I said, well forget 2, now you have 3 to 10: what's your choice? And they all said 4. In this way the number order turned out to be: 2, 4, then 8, 3 and 6 lumped together (there was some dispute about their order). Then came 9 and 5 (some dispute here again), followed last of all by 7. Look at the indigestibility values: you'll find a very similar rating.

Another case was a Stanford University experiment of 1975, in which people subjectively evaluated similarities of the digits 0 to 9. They found that if a computer placed these digits on a sheet of paper, such that their distance would match the similarity ratings (a technique called multi-dimensional scaling), the digits increased in magnitude from left to right - see Fig.10. Now, the computer didn't know these were numbers, just took them as symbols! Note also that the even numbers are separated from the odd by a horizontal line. Another slanting line separates prime numbers from composites, a line I found to be practically my indigestibility axis: by plotting perpendiculars to that, I found it comes pretty close to the indigestibility formula - another case where I was gratified to find a parallel in nature. Well let's call it nature!

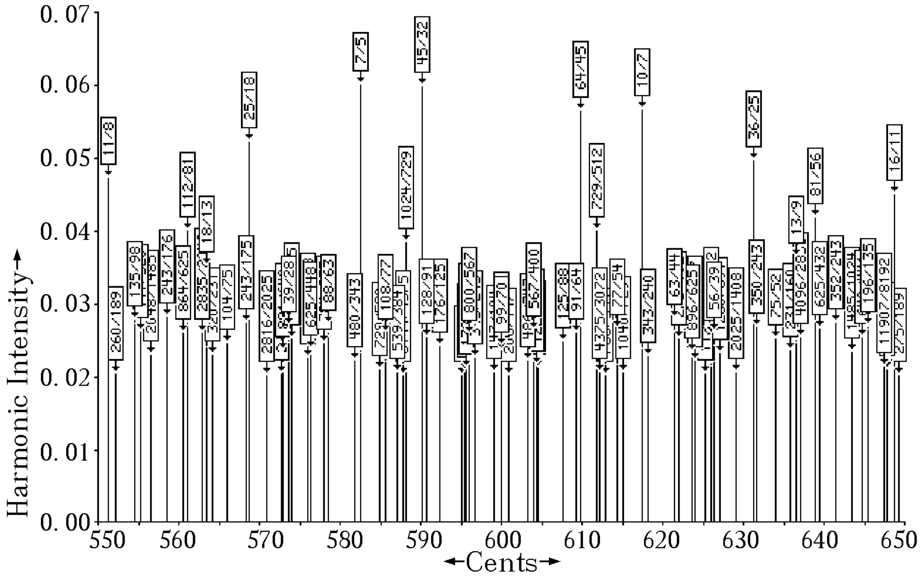
Fig.10 - Numerical Similarity Scaling (Stanford 1975)



Using the harmonicity formula I then proceeded to rationalise scales. It's all very well to say, the more harmonic the interval the more probable it should be in a music, but how do I know that the perfect fifth (shall we say) is a harmonic interval? By listening to it maybe. I could evaluate each interval by listening. But there are cases where I wouldn't be very sure, for example in the case of micro-intervals. The piece we heard, *Otodeblu*, had 17-tone tuning, with lots of intervals I'd never known before, never learnt about in school. How would I know their ratios? Because if you talk about a perfect fifth, for instance, you're talking of a scale degree, not a ratio. How do I know the perfect fifth could be a 2:3? How could I know the ratio for a neutral third of 350 cents?

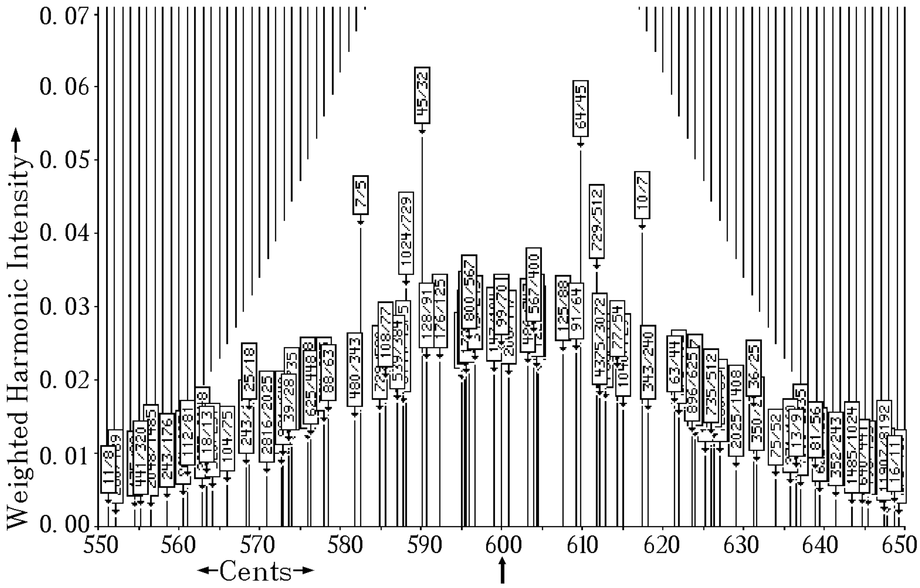
Some systematic method of rationalisation would have to be developed: I decided to go about it as follows. Let's look for instance at the region around the tritone, 600 cents (Fig.11a) within a range of plus or minus one quarter tone. I collect a large number of intervals in this range and plot them according to their pitch and their harmonicity. Here, for example, is a 45/32, a 64/45, a 10/7, a 7/5 and a whole lot of others very densely packed together[2]. Which one of these several ratios could be a given tritone?

Fig.11a - A list of several intervals between 550 and 650 cents



The procedure I used was to put a Gaussian bell-shape over the place that I want to tune, to rationally understand (see Fig.11b. - the bell-top is clipped). Now only a few candidates are left, those further away being pushed out of existence. The width of this bell is variable, depending on your own intervallic tolerance.

Fig.11b - The same intervals, harmonically weighted in favour of 600 cents



What do I do with these, shall we say two or three best candidates?

Let's repeat this for the major scale in Fig.12a.: here is an octave, with the perfect fifth 3/2 standing very high there in the middle, the perfect fourth 4/3 further left. Plus-minus signs are ignored.

We place bells over the places to be understood (see Fig.12b). In each tent-like enclosure, we see some candidates left over. Taking the two or three best candidates, we tune the whole scale by using candidate 1 of degree 1, candidate 1 of degree 2 and so on. Then candidate 2 of degree 1 against candidate 1 of degree 2 and so on. For every possible combination of a candidate per scale degree, we add all the intra-intervallic harmonicities - the sum indicates how harmonic the general tuning is. And as a matter of fact the result comes out very nicely.

Fig.12a - A list of 77 intervals in an octave

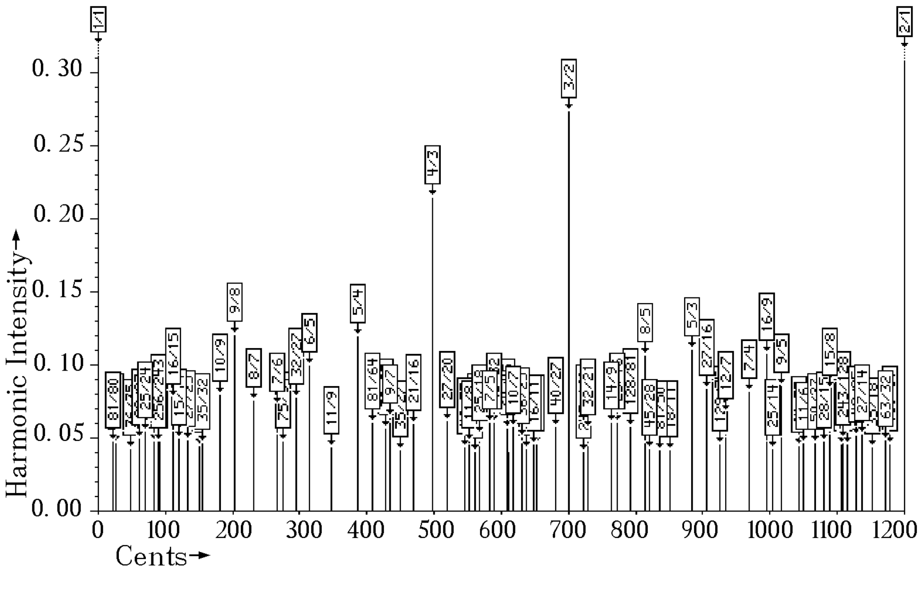
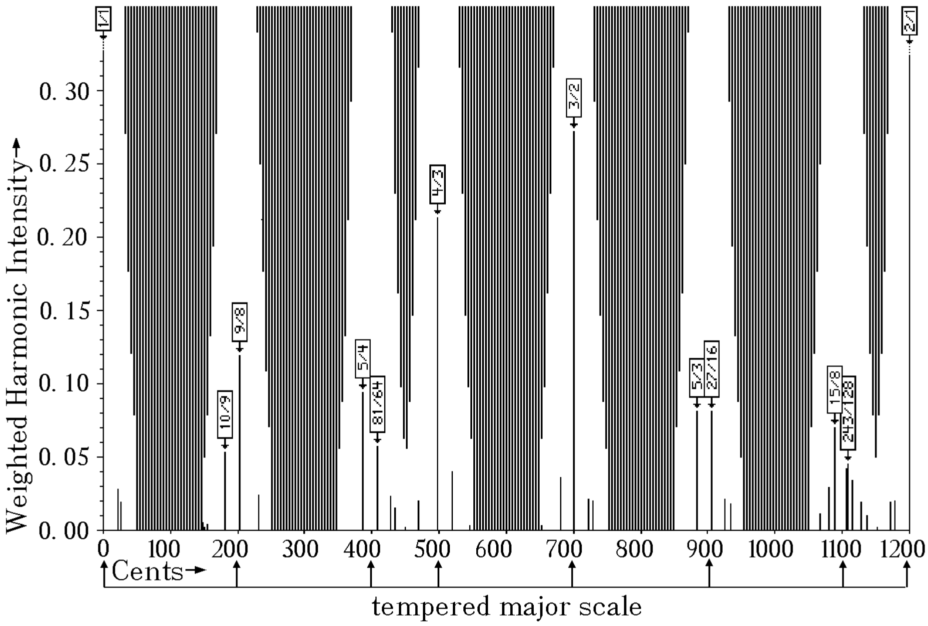


Fig.12b - Ditto, weighted for a tempered major scale (bell-tops clipped)



Doing this to a twelve-tone chromatic scale, the rationalised set

1:1, 15:16, 8:9, 5:6, 4:5, 3:4, 32:45, 2:3, 5:8, 3:5, 9:16, 8:15, 1:2.

is produced, which according to theory books, is the classical tuning of the harmonic chromatic scale.

Doing the same for a 13-tone tuning (0, 92, 185 cents etc.) we get

1:1, 128:135, 8:9, 6:7, 4:5, 16:21, 35:48, 56:81, 160:243, 5:8, 7:12, 5:9, 128:243, 1:2,

intervals we actually know quite well.

Finally, a 17-tone tuning (0, 71, 141 cents etc.) yields

1:1, 24:25, 25:27, 8:9, 27:32, 9:11, 25:32, 3:4, 18:25, 25:36, 2:3, 16:25, 11:18, 16:27, 9:16, 27:50, 25:48, 1:2

I've tried this on Indian and Arabian scales with plausible results.

J.Tenney: Clarence, how wide is your tolerance?

C.Barlow: Here 30 cents nominal tolerance - the place at which the Gaussian Bell arbitrarily reaches one 20th of its maximum. I usually use about half the smallest interval.

J.Tenney: So what is the cents value for which you arrived at 6:7?

C.Barlow: 277 cents as input; the output is 10 cents lower.

J.Tenney: So you've filtered out 5:6 by your Gaussian.

Fig.13a - Rational 12-equal (h>0.115):

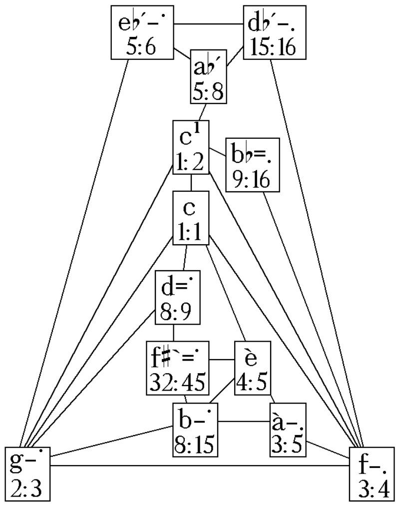
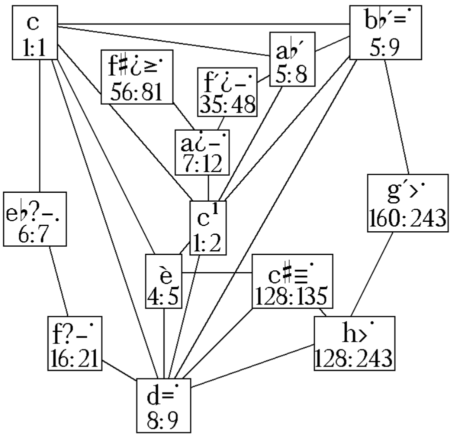


Fig.13b - Rational 13-equal (h>0.072):

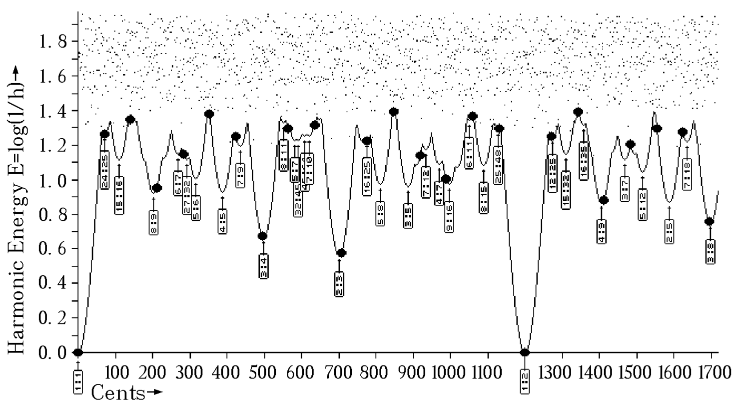


C.Barlow: Exactly - it all depends on your tolerance! The result here deviates from the input by less than 17 cents. Fig.13a is a diagram showing the network for a twelve-tone tempered system of all intervals more harmonic than 0.115. You find the tonic [C] in the middle linked with [D] and [G] but not with [F♯]. But [F♯-B], [B-G], [B-A], [F-D♭], [D♭-E♭] are all linked. Below the note-names are the ratios of these intervals.

In Fig.13b, the same is done for the thirteen-tone tempered system. I've given them names like these, derived from whether they're tertian or septimal intervals (based on the primes 5 or 7), but the ratios might mean more to you.

Fig.14 shows work of my friend and former student Georg Hajdu, now in Berkeley, California[3]. Based on my harmonicity formula, it concerns what he calls the "energy of a pitch space". You see downward bulges or dips at the more harmonic intervals. To rationalise a 17-tone tempered scale, he puts 17 equidistant balls into an octave of this pitch space and sees where they appear. In many cases they're on the flank of a dip. In such cases, he says, there is a strong pull on the ball causing a rationalisation of the scale degree into the dip. His method plots an energy curve and finds where the balls want to fall in. It's interesting to compare this rationalisation method to my own, candidate-based one.

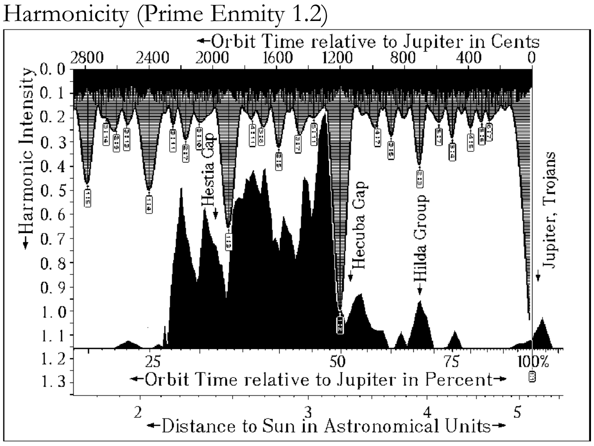
Fig.14 - A 17-equal scale against a map of Harmonic Energy (after Hajdu)



4. Harmonicity in the Sky

Talking about curves and balls falling into things... Here's something completely different: a map of density of the asteroid belt between Mars and Jupiter (Fig.15)...

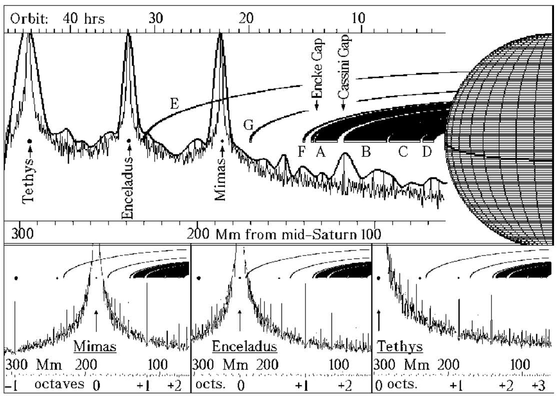
Fig.15 - Relative Density of Asteroid Belt vs. Harmonicity (Prime Enmity 1.2)



There are practically no asteroids around Mars, which is way over on the left outside the diagram. And on the right at Jupiter you have the Trojans, occuping the same orbit. The gaps you can see are caused, according to astronomers, by the "commensurability" (harmonic ratios) between the places in question and the planet Jupiter in relation to their orbiting time around the sun. The place marked 100% is at Jupiter's orbit. Look carefully at the 50% mark (1:2, a "distance octave" from Jupiter, so to speak) - you see a very deep cleft there called the Hecuba gap. Another one is at 33% (1:3). These gaps ought to and are indeed comparable to the harmonicity landscape running from left to right: the harmonicity increases practically everywhere there's a gap. These values are based here on a modified indigestibility formula which is prime friendlier: the power ("prime enmity factor") was experimentally lowered from 2 to 1.2, so it's the same formula, but with the power changed.

Here is an attempt to investigate the commensurability gaps in the rings of Saturn, based on harmonicities from the orbiting times of its satellites (Fig.16), of which the first three, Mimas, Enceladus and Tethys are individually shown in the lower half of the diagram; in all these curves, the harmonicity value is multiplied by the cube root of the mass and divided by the distance from a given satellite. The harmonicity curves generated by all three show peaks corresponding to the big Cassini gap between the A-and B-rings. The combined curves of the eight largest satellites is shown in the upper half.All this was done out of curiosity. And it matches rather nicely.

Fig.16 - Saturn rings A-G, harmonicity curves from satellites



So much about harmonicity. My measurement of intervallic harmonicity, the results of which I found very plausible, enabled me to compose a certain type of music: given a scale in cents, I was able to rationalise it and then to create fields of variable tonality by altering the probability of intervals according to their harmonicity. The question of scale has occupied me for a very long time. I'll make a couple more observations on this point in a minute. But now I'm going back to metre for a while.

5. On Metric Affinity[8]

I spoke earlier about the indispensability of various pulses, i.e. of how relatively important pulses were. In 1981 I tried for the first time to use this method to find a measure of the Affinity of two metres, of how similar they might be felt to be. Suppose I match two metres in such a way that their pulses lie one against one. And if they don't match, I subdivide the pulses suitably and sufficiently so that their finest subdivisions do. And then I measure the indispensability of each pulse.

Here I have a 2x2x3 pulse system and a 3x5 pulse system (Fig.17); the indispensability values [11,0,4,8,2,6,10...] of the 2x2x3 metre are matched against the [14,0,9,3,6,12...] of the 3x5 metre.

Fig.17 - Pulse Indispensabilities for 2x2x3 against those of 3x5

Current Pulse:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2x2x3 Pulses:	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8
Indispensability:	*11*	*0*	*4*	*8*	*2*	*6*	*10*	*1*	*5*	*9*	*3*	*7*	*11*	*0*	*4*	*8*	*2*	*6*	*10*	*1*
3x5 Pulses:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5
Indispensability:	*14*	*0*	*9*	*3*	*6*	*12*	*1*	*10*	*4*	*7*	*13*	*2*	*11*	*6*	*8*	*14*	*0*	*9*	*3*	*6*
Current Pulse:	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
2x2x3 Pulses:	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4
Indispensability:	*5*	*9*	*3*	*7*	*11*	*0*	*4*	*8*	*2*	*6*	*10*	*1*	*5*	*9*	*3*	*7*	*11*	*0*	*4*	*8*
3x5 Pulses:	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10
Indispensability:	*12*	*1*	*10*	*4*	*7*	*13*	*2*	*11*	*5*	*8*	*14*	*0*	*9*	*3*	*6*	*12*	*1*	*10*	*4*	*7*
Current Pulse:	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
2x2x3 Pulses:	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
Indispensability:	*2*	*6*	*10*	*1*	*5*	*9*	*3*	*7*	*11*	*0*	*4*	*8*	*2*	*6*	*10*	*1*	*5*	*9*	*3*	*7*
3x5 Pulses:	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Indispensability:	*13*	*2*	*11*	*5*	*8*	*14*	*0*	*9*	*3*	*6*	*12*	*1*	*10*	*4*	*7*	*13*	*2*	*11*	*5*	*8*

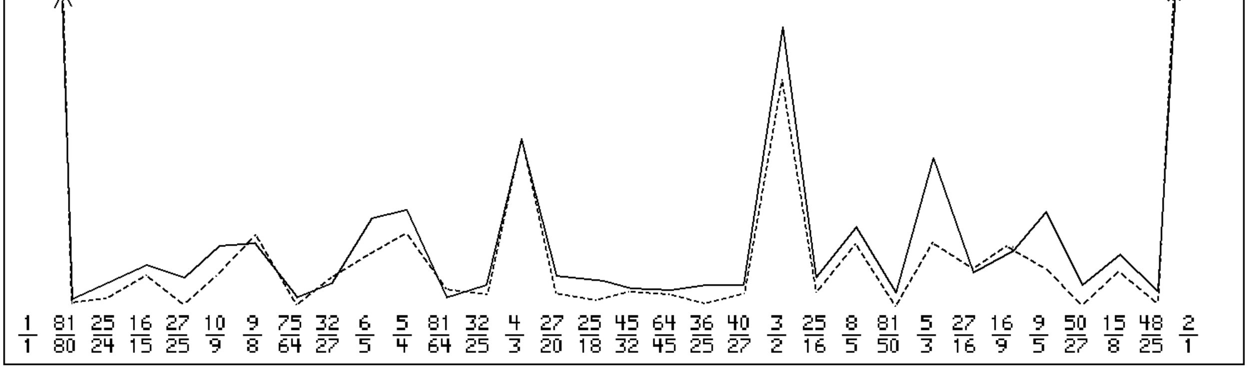
Fig.18 - Metric Affinity formula[1]

$$M = -\frac{1}{2\log_e\left(\dfrac{18\sum_{n=1}^{\Omega_0}\left[\prod_{i=1}^{2}\left\{\psi_{z_i}\left(1+(n-1) \bmod \Omega_{z_i}\right)\right\}\right]^2-2}{7\Omega_0\prod_{i=1}^{2}(\Omega_{z_i}-1)^2}\right)}$$

Multiplying the corresponding indispensabilities of the two metres, squaring the products and adding the squares, I arrived at a formula (Fig.18) for the metric affinity and found different affinity values for various pairs of metres.

I then took simple ratios like 1:2, 2:3 as if they were polymetres. For 2:3, imagine one bar with two pulses and one equally long in time with three, and measure their metric affinity. The joined line in Fig.19 is the metric affinity curve, compared to the dotted line, the harmonicity curve for the ratios. I was flabbergasted to find two totally different methods yielding results so unexpectedly similar.

Fig.19 - Metric Affinity (joined line) vs Harmonicity (dotted) for simple ratios



6. On Ragas

I'll go back now to 1967, when I lived in Calcutta and began to learn about Indian music, about which I'd known nothing before except some names of instruments, like sitars, but which I'd not listened to consciously. My family was anxious to preserve its feeling of Angloid identity, and it really wasn't the done thing to listen to such music. The day I brought my first sitar, the hue and cry there was in the family! And the first time I wore Indian clothes! But I managed to get my parents and other relatives used to it - little by little.

I was nineteen when I began to actively listen to Indian music; and I discovered that there was quite a lot about it that I could learn quickly through my knowledge of European music. So I began to study the structure of various scales and ragas[4].

Soon after - this is already about twenty years ago - I combined standard Indian tetrachords to form a table of modes shown in Fig.20 in Indian notation *Sa Re Ga Ma Pa Dha Ni Sa* (meaning exactly Do Re Mi Fa Sol La Ti Do) in abbreviated form: here the small letters "..rgm.." mean minor intervals plus perfect fourth, the capitals "..RGM.." major intervals plus augmented fourth. I enjoyed inventing Pseudo-Grecian names for the tetrachord combinations, names like Phrylyrian, Miphryxian, Iophrynigian, or Dolian, Lylian and so on. You also find here all the church modes Dorian, Phrygian, Lydian etc. (the Mixolydian is here termed "Mixian"). But this was also a precise nomenclative handle; e.g. the syllables "-phry-di-" imply a minor second with an augmented fourth and a major seventh, combining Phrygian and Lydian.

D.Lekkas: It also means an eyebrow.

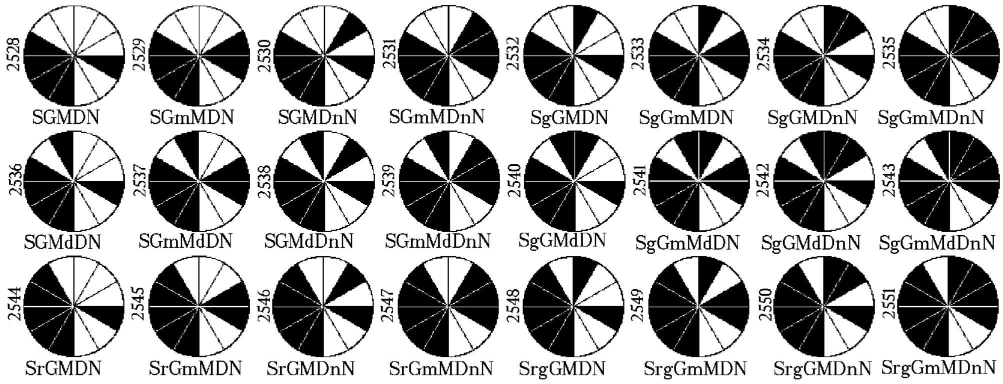
C.Barlow: Does it? Well one could raise that! ....

Fig.20 - The Barlopoulos Pseudogrecian Nomenclative System for the Forty-Eight Septagenous Heptatonic Scales[5]

	pdnS ´	pdNS ´	pDnS ´	pDNS ´	PdnS ´	PdNS ´	PDnS ´	PDNS ´
Srgm	Locrian	Lonicrian	Locririan	Locrinian	Phrygian	Phrynigian	Phryrian	Phrynian
SrgM	-------	-------	-------	-------	Phryligian	Phrylydigian	Phrylyrian	Phrylydian
SrGm	Milocrian	Iolonicrian	Milocrixian	Iolocrinian	Miphrygian	Iophrynigian	Miphryxian	Iophrynian
SrGM	-------	-------	-------	-------	Lyphrygian	Lyphrydigian	Lyphryxian	Lyphrydian
SRgm	Æocrian	Æonicrian	Docririan	Docrinian	Æolian	Æonilian	Dorian	Donian
SRgM	-------	-------	-------	-------	Æolylian	Æolydilian	Dolyrian	Dolydian
SRGm	Micrian	Ionicrian	Micrixian	Iocrinian	Milian	Ionilian	Mixian	Ionian
SRGM	-------	-------	-------	-------	Lylian	Lydilian	Lyxian	Lydian

Around the same time, I mapped possibilities of scale structures with a computer - see Fig.21. Each segmented circle here is a scale, the segments being not of a cake or a pizza, but adjacent notes in an enharmonic twelve-tone cycle[6] of fifths - filled segments mean notes present. Some scales have a whole bunch of adjacent notes; if there are e.g. five, they form a regular pentatonic scale, opposite to which there could also be one or two more adjacent notes. Suppose I take the five black piano keys and add a perfect fourth, say [A+D] to them as a drone: this is scale #2545 in the diagram; it confronts a tonic-subdominant drone with a foreign pentatonic scale. The harmonic potential can thus be predicted by looking at the diagram. And this kind of harmony is indeed used in ragas.

Fig.21 - Twenty-four scales depicted as 12-bit clockwise cycles-of-fifths: the tonic (coded as the most significant bit) is at 3-4 o'clock.



Two aspects of ragas fascinate me especially, which I've not found in any book. One I've just mentioned; the other's a set of "one-way streets". Here's a tape of ragas *Behag* and *Kedar* with three pieces each. Both ragas have the same notes (major scale plus augmented fourth); what differs is the typical melodic pattern, shown here on tonic [C] -

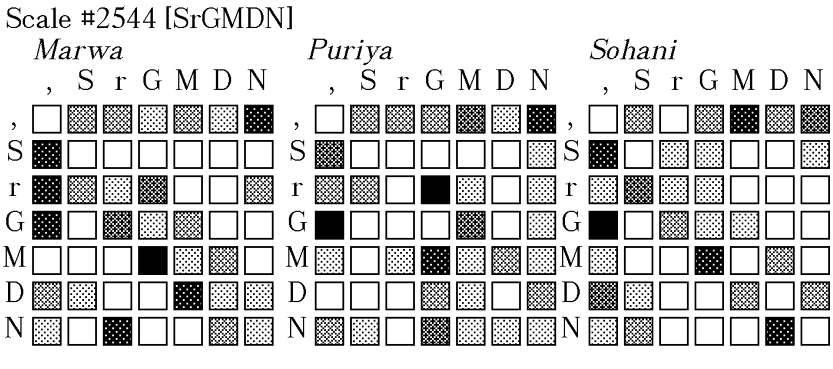
Behag: C E F G B c B, (A) G F♯ G E F♮ E, (D) C

Kedar: C F, E G, F♯ A G c, A G F♯ A G F♮, D C

[Tape examples of musicians: Ragas *Behag* & *Kedar*]

This note-routing is one of the aspects that makes ragas special. One can draw diagrams of the probability that a note in a given raga will go to another note (a so-called Markov chain of order 2), as in Fig.22 : three different ragas use the same scale (#2544). The diagrams show that the ragas are distinctly different.

Fig.22 - Statistical behaviour of three isoscalic ragas - density of shading indicates relative frequency of occurrence



With this understanding I typed various raga patterns into the Cologne University main computer in 1976 and synthesized Markov chains, in other words new patterns in the same ragas. Then I played these syntheses to musicians in Calcutta. Now some ragas I believed would be recognizable only from Markov level 2 on - i.e. considering at least two-note patterns. But some would be already clear on level 1. And some, raga *Gaursarang* particularly, would need level 3. This suspicion was indeed completely borne out by my experiment. It's something I'll write about in detail some day.

Twenty years ago I started a catalogue of ragas by coding the presence or absence of a given note in the octave, a simple bit configuration. The presence of a *Sa* (the tonic) would yield 2048; you set the various bits for each scale degree as a member of an enharmonic cycle of twelve fifths, getting a binary code. I had this exercise book in which I used to, whenever I heard a raga new to me, enter it at its code number. There were of course lots of gaps, because not every combination is used. But there are some places where there were so many entries, I had to stick an extra page on. For example #4035 is the major scale with the added augmented fourth - there are no less than fifteen ragas listed here. It's one of the most popular scales in Indian music.

The one-way street system is the second aspect I mentioned. Let's go back to the first aspect, that of a raga's harmonic possibilities. A raga using the black keys added to [D] and [G] as drone would match #2297 in my catalogue ([SrGmMdN] - not shown in Fig.21) and could - and would - be played as raga *Lalit* by musicians.

[Tape example: Raga *Lalit*]

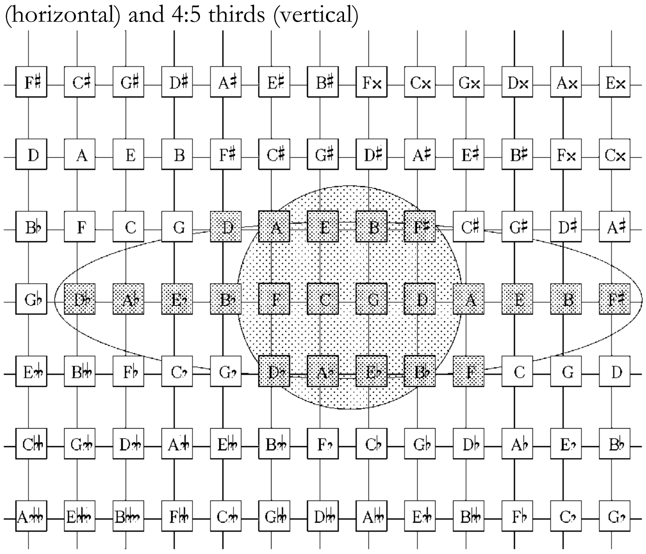
In fact if you take all combinations of five black keys placed on any perfect fifth, with either of the fifth's two notes as tonic (e.g. if you have [G] and [D], either could be the tonic), you would get 24 possible ragas. And this is borne out by practice.

7. On the Origin of Scales

Talking about scales always presumes you have a set of pitches to go by. But what makes these pitches arise? What if you want to devise a scale from a different set of prime factor ratios? If the prime number 7 is to help generate a scale, what do you do?

Take the familiar grid of fifths and thirds to be found in theory books (see Fig.23); those who use it know that the chromatic scale's twelve notes forms a kind of squat rectangle in the middle. The third being less harmonic than the fifth, I stretched the vertical axis to counterbalance this disadvantage[7]: now the chromatic scale is in a nice circle - which seems to make sense. The 22 srutis of Indian music are enclosed in an ellipse -they're in shaded boxes:

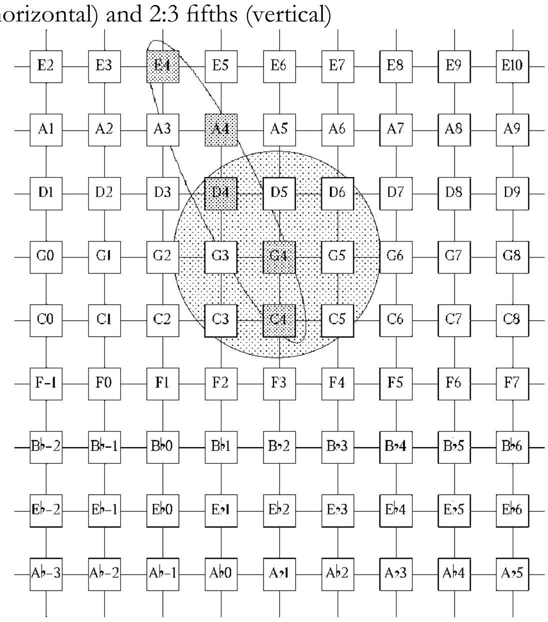
Fig.23 - Network of 2:3 fifths (horizontal) and 4:5 thirds (vertical)



Is this how scales are generated? If you throw an oil film on a pitch grid, stretched or not, and it forms a nice round shape, would this contain a prospective scale? I think not!...

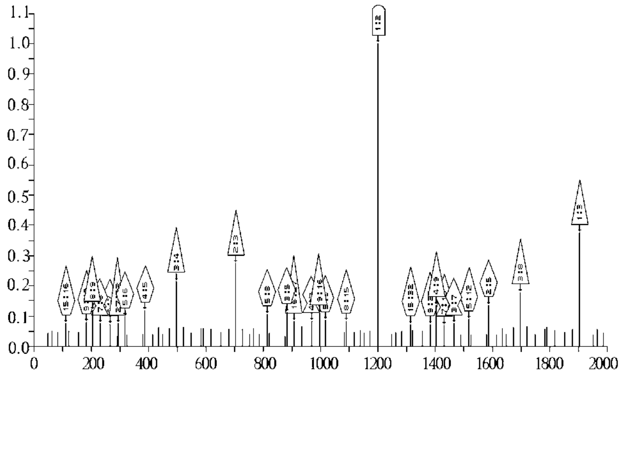
Look at the grid of octaves and fifths in Fig.24 - normally, the poor old octave is always discriminated against, always taken for granted. But it's an interval in its own right. A scale like the encircled one - [C3 G3 C4 D4 G4 C5 D5 G5 D6] - would hardly be useful. It would seem that the nearness of the notes will have to play a role as well and that this grid is nothing more than just a representation, a diagrammatic interval catalogue because as you see it doesn't yield a usable scale. The pentatonic scale, commonly supposed to consist of octaves and fifths, is enclosed by an implausibly longish cigar-like shape imaginable on the upper left, written in shaded boxes (interesting, though, all these ellipses!):

Fig.24 - Network of 1:2 octaves (horizontal) and 2:3 fifths (vertical)



I'm beginning to move in the following direction (but haven't got far yet). Imagine, for example, a 3-limit system of harmonicity, (i.e. based on the prime numbers 2 and 3) and limit this to 2000 cents (nine intervals in triangular boxes in Fig.25), but of course you can go much beyond this. And cast a kind of filtering Gaussian curve somewhere in the centre of gravity of all the pitches you've got. That might yield a scale. In the diagram, all intervals more harmonic than an arbitrary threshold of 0.07 are shown as ratios; twelve 5-limit (primes upto 5 in pentagonal boxes) and five 7-limit intervals (in heptagonal boxes) are also shown. These intervals all commonly occur in scales of numerous world cultures. Raising the harmonicity threshold to 0.107 would cause sixteen of the intervals to be filtered out, leaving a pure Mixolydian scale.

Fig.25 - List of intervals≤2000 cents marked according to prime-limit



I could therefore imagine trying to develop scales out of prime number material - not by drawing circles or ellipses on a grid, but by setting a harmonicity threshold and selecting intervals of all possible configurations within a certain prime limit, for example 7. And by possibly drawing a Gaussian curve somewhere in there and seeing what gets filtered out. That might be a usable scale.

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1. See Computer Music Journal Vol 11 Nº 1 (1987) for more detail.

2. In this text, a ratio notated with a slash (/) indicates the direction of the corresponding interval, e.g. 4/5 is a falling third, 5/4 a rising one. A colon (:) ignores the direction, as in 4:5.

3. Georg Hajdu now lives in Münster, Westphalia (Author's note 2007: Georg Hajdu is now Professor of multimedia composition and music theory at the Hamburg Hochschule of Music and Theater; see <http://www.georghajdu.de/>).

4. The word is pronounced "ra:g"; I used to write it "raag" to counteract common mispronunciation, but you can't always swim upstream!

5. The term "septagenous" pertains to seven classes of pitches (*sa*, *re*, *ga*, *ma*, *pa*, *dha*, *ni*), each - with the exception of *sa* - manifest in two forms, the lower *komal* (rgmpdn) and the higher *tivra* (RGMPDN). The complete range of pitch names S/rR/gG/mM/pP/dD/nN is exactly equivalent to theWestern nomenclature C/DbD/EbE/FF#/GbG/AbA/BbB (here arbitrarily C-based). Though it is theoretically known, komal pa ("Gb") is never referred to in practice, since it does not coexist with the preferred tivra ma ("F#") in scales of Indian music.

6. These 4095 possible scale-circles (2048 of them containing a tonic) can be reduced to 351 scale-cycles in all, 66 of which are heptatonic - of which one is the cycle of seven church modes (Ionian, Dorian, Phrygian, Lydian, Mixolydian, Aeolian and Locrian).

7. The stretch is effected here by the factor $Z_\gamma(P)/Z_\gamma(Q)$, where P/Q represents the interval of the Y-axis and $Z_\gamma(n) = 1 + \log(256)/\log(27)$ when n=2, else $= 2(n-1)^\gamma/n + \log(n)/\log(2)$.. γ is the "prime enmity factor", set here at 1.2 for Fig.23 (interval 5/3, stretch 1.422) and Fig.24 (3/2, stretch 1.162).

8. Author's note 2007: For reasons explained in my yet unpublished book, I now call this attribute "Metric Coherence".